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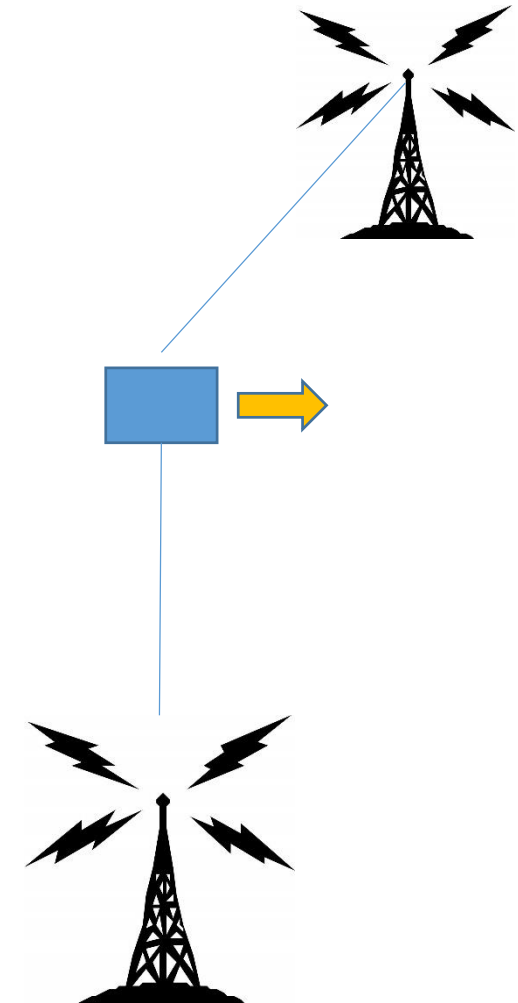
# Correlation-agnostic Fusion for Improved Signals of Opportunity-based Navigation

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# RF-based Velocity Estimation

- Assume known location of emitters (GNSS, Signals of opportunity, etc.)
- Doppler shift from each signal can be used to create a 1d “velocity vector”
- Multiple vectors used to create overall velocity estimate
- Base frequency of each emitter depends on local clocks
  - Clocks may be correlated -> correlated errors between velocity measurements
  - Do not know correlation at estimator



# Does Correlation Matter? (1/3)

- Outputs must include more than just the “best estimate”
  - Requires uncertainty information
- Accurate uncertainty matters
  - Example 1: Geo-location
  - Example 2: Path planning
    - Autonomous system desires to drive between two buildings 10m apart.
    - Is uncertainty of position 1m or 20m?

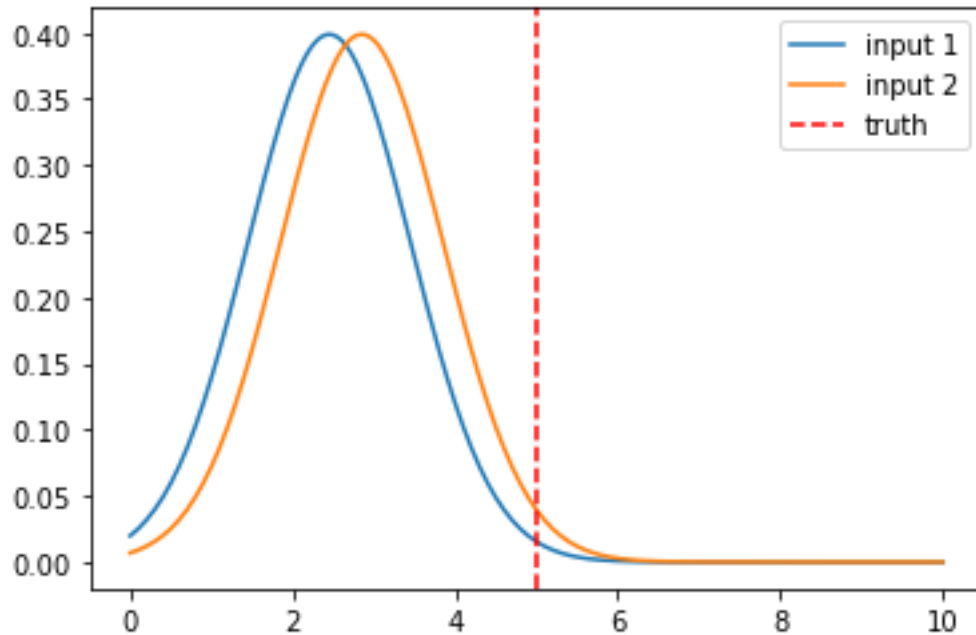


Something of interest is here...

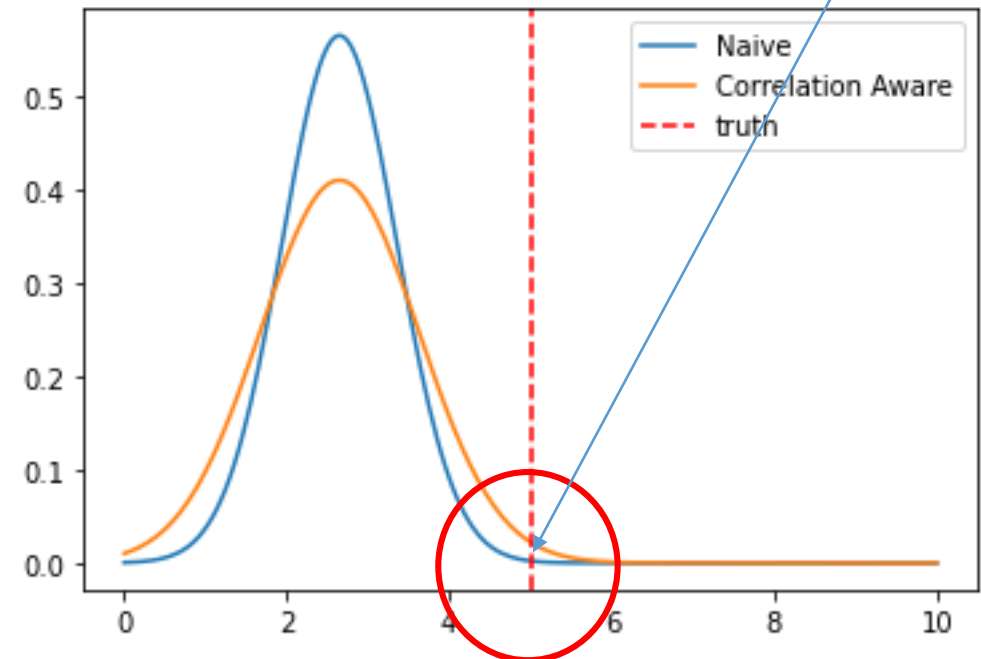
# Does Correlation Matter (2/3)

- Correlated errors can lead to:
  - Significant Over-confidence
  - Incorrect estimates
- Example:

Input PDFs (highly correlated)



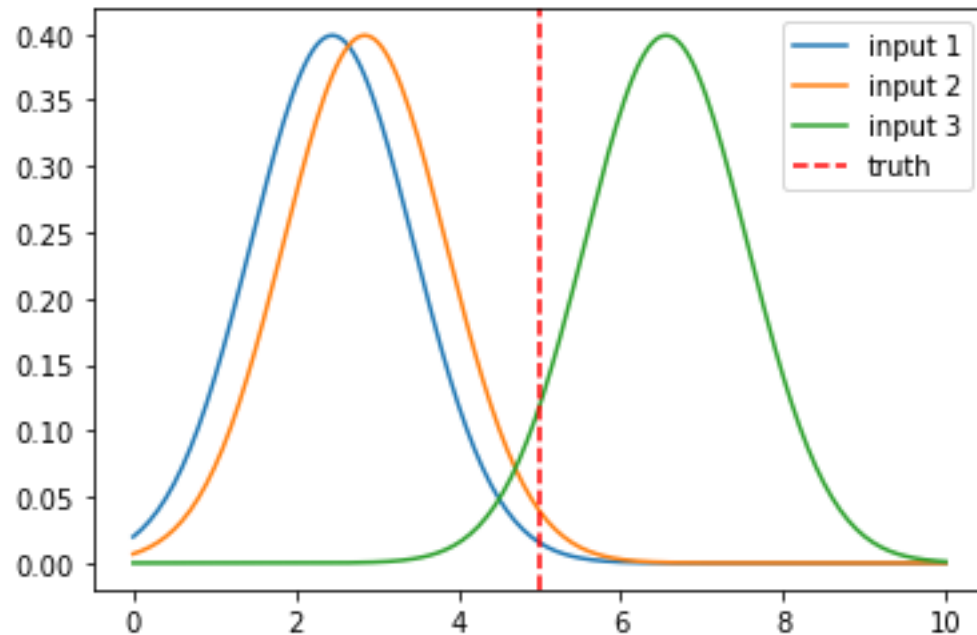
Fused PDFs



# Does Correlation Matter (2/3)

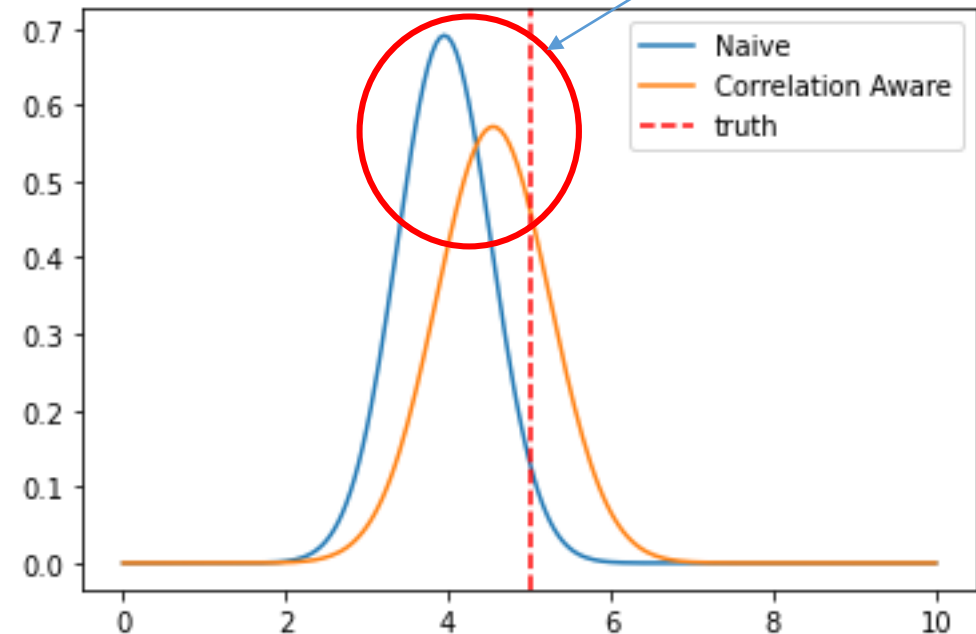
- Correlated errors can lead to:
  - Significant Over-confidence
  - Incorrect estimates
- Example:

Input PDFs (two are highly correlated)



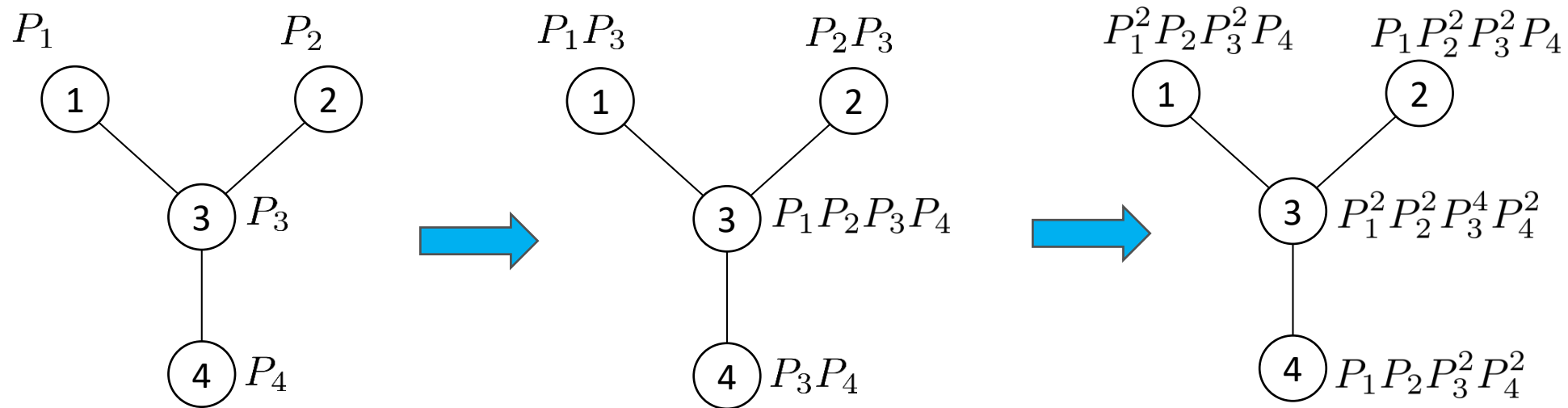
Knowing two left inputs are correlated leads to more accurate state estimate

Fused PDFs



# Why not just use the correlation information?

- Scenario 1: Distributed estimation in large network
  - Track to track correlation problem

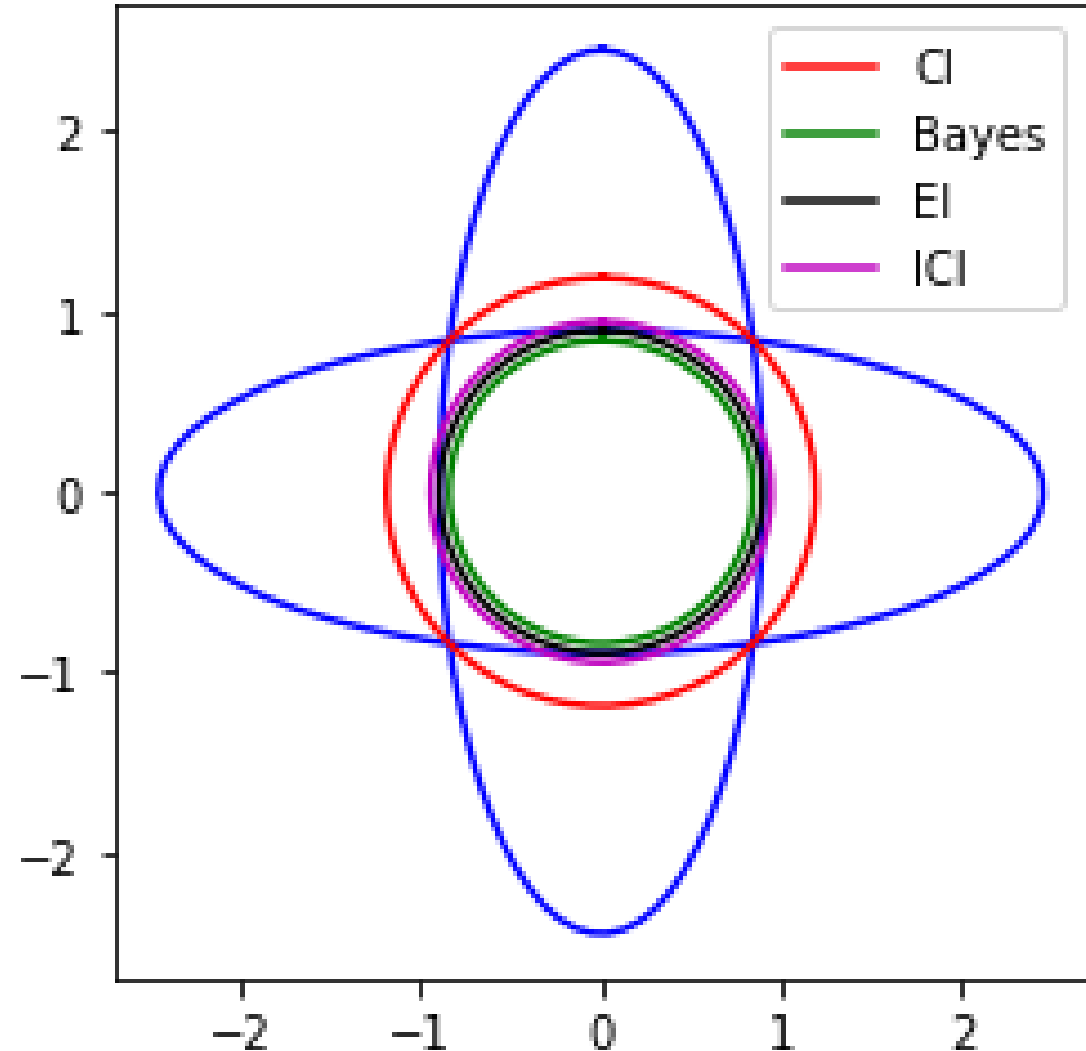


- Scenario 2: Signals of opportunity

- Clocks may be synchronized -> correlated errors
  - Differences between GNSS systems
  - Ground emitters may be synchronized to a particular clock source

# Correlation-agnostic Fusion

- Prior techniques:
  - Covariance Intersection –
    - Find ellipse that bounds common covariance
  - Ellipsoidal Intersection
  - Inverse Covariance Intersection
- All these techniques use only the covariance of the inputs to determine amount of correlated information



# Probabilistic Constraint

- Problem setup

$$P_a = P_{a \setminus c} P_c$$

$$P_b = P_{b \setminus c} P_c$$

$$P_f = P_{a \setminus c} P_{b \setminus c} P_c$$

- Key Insight:

- For Gaussian distributions, means should all be within statistical bounds of each other

- We can test for if means came from same distribution

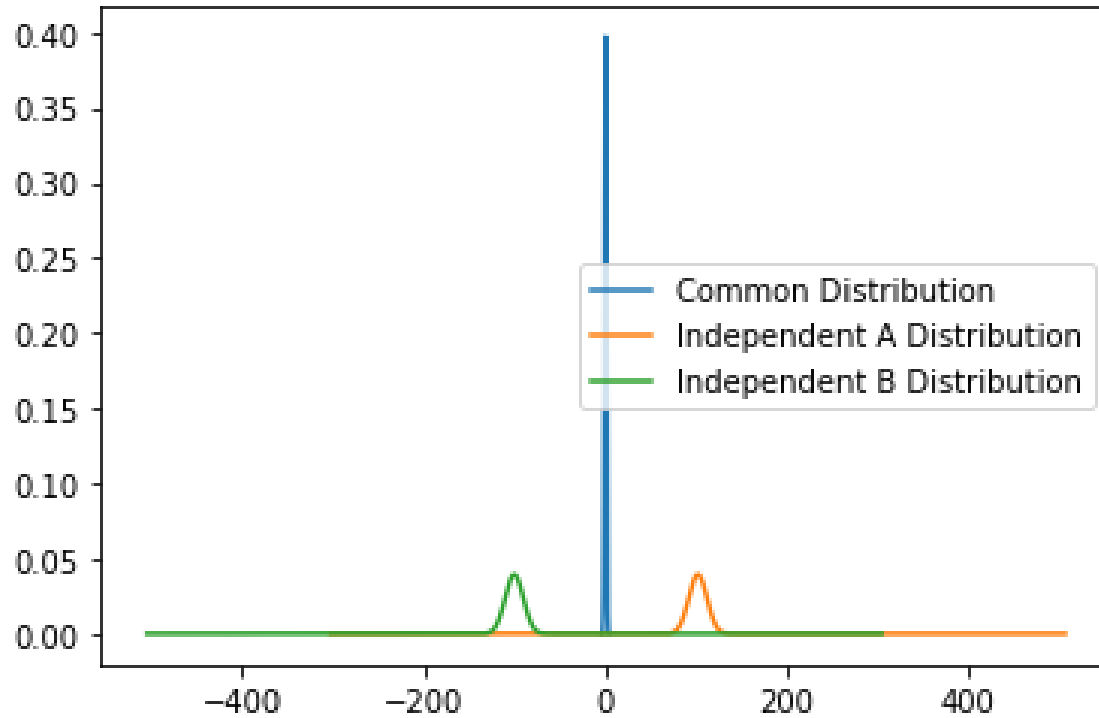
- Mahalanobis distance between independent distribution is a statistical null hypothesis test of means' being from same distribution

$$\Psi(C_c) = (\mu_{a \setminus c} - \mu_{b \setminus c})^\top (C_{a \setminus c} + C_{b \setminus c})^{-1} (\mu_{a \setminus c} - \mu_{b \setminus c})$$

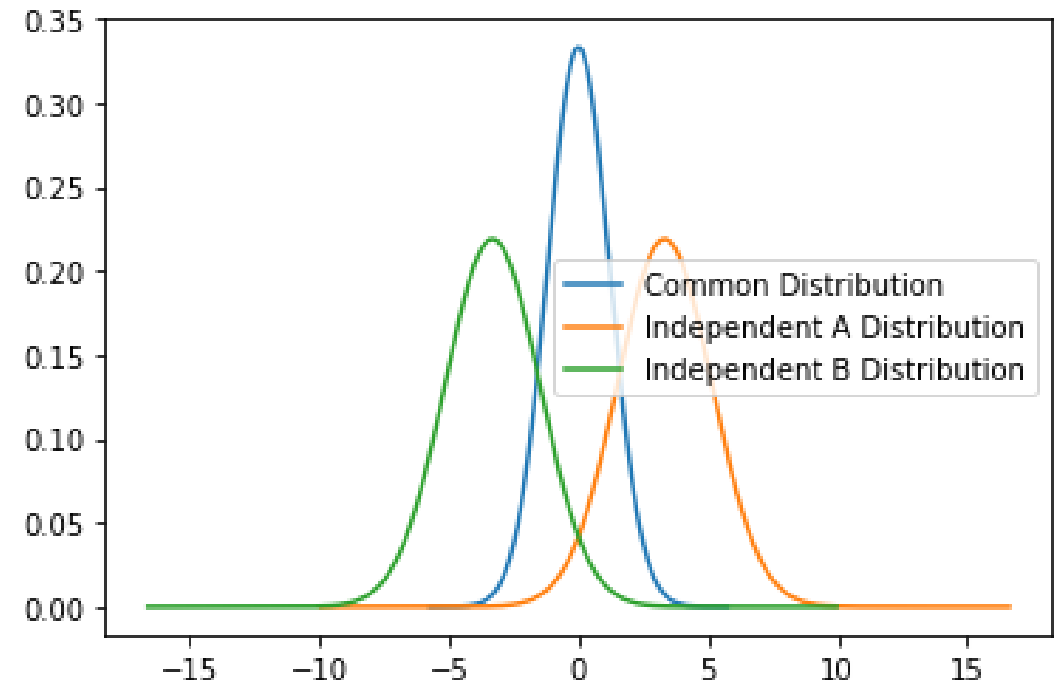


# Input Distribution Covariances Equal

- Input Distributions:  $(1, 1) \mid (-1, 1)$ 
  - EI computed:  $\Psi = 100.5$
  - Fused EI covariance: 1
  - PC constrained:  $\Psi = 6.6348$
  - Fused PC covariance: 0.77



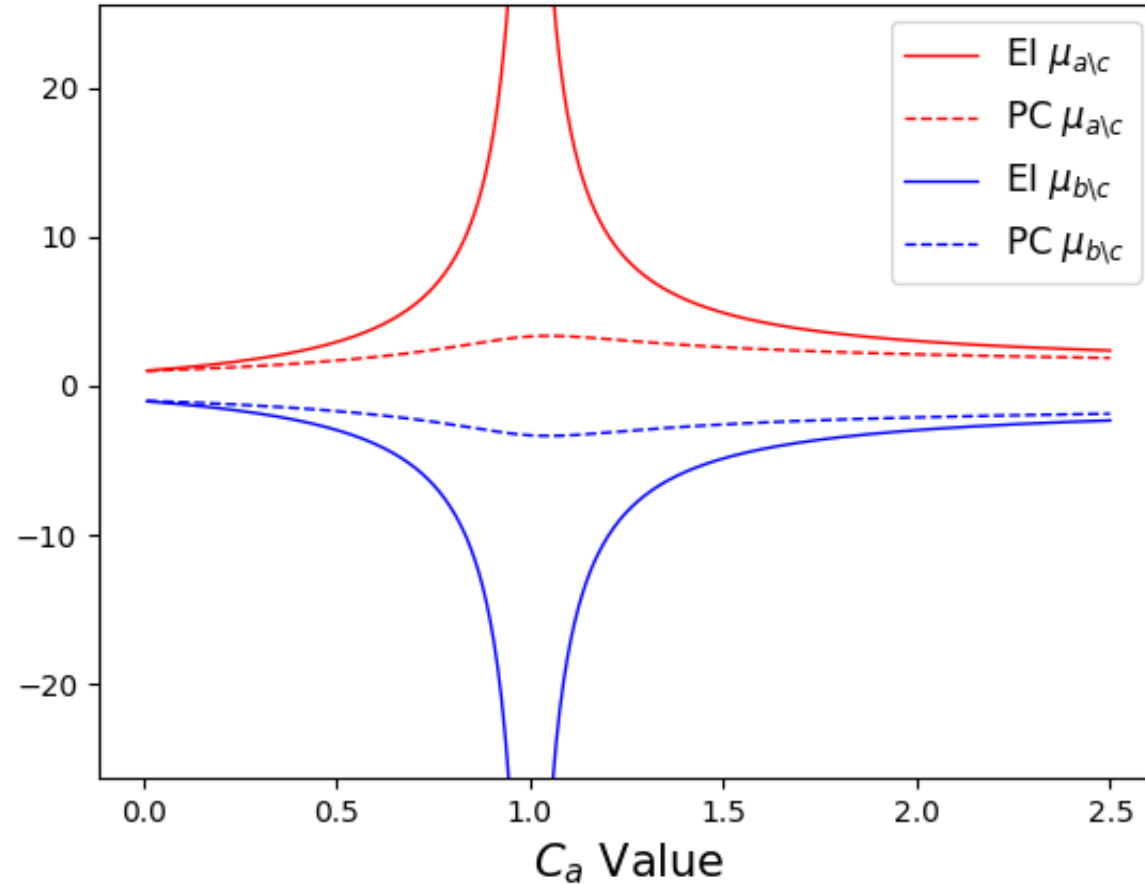
Ellipsoidal Intersection



Probabilisticly Conservative

## Case Study: Varying Covariances

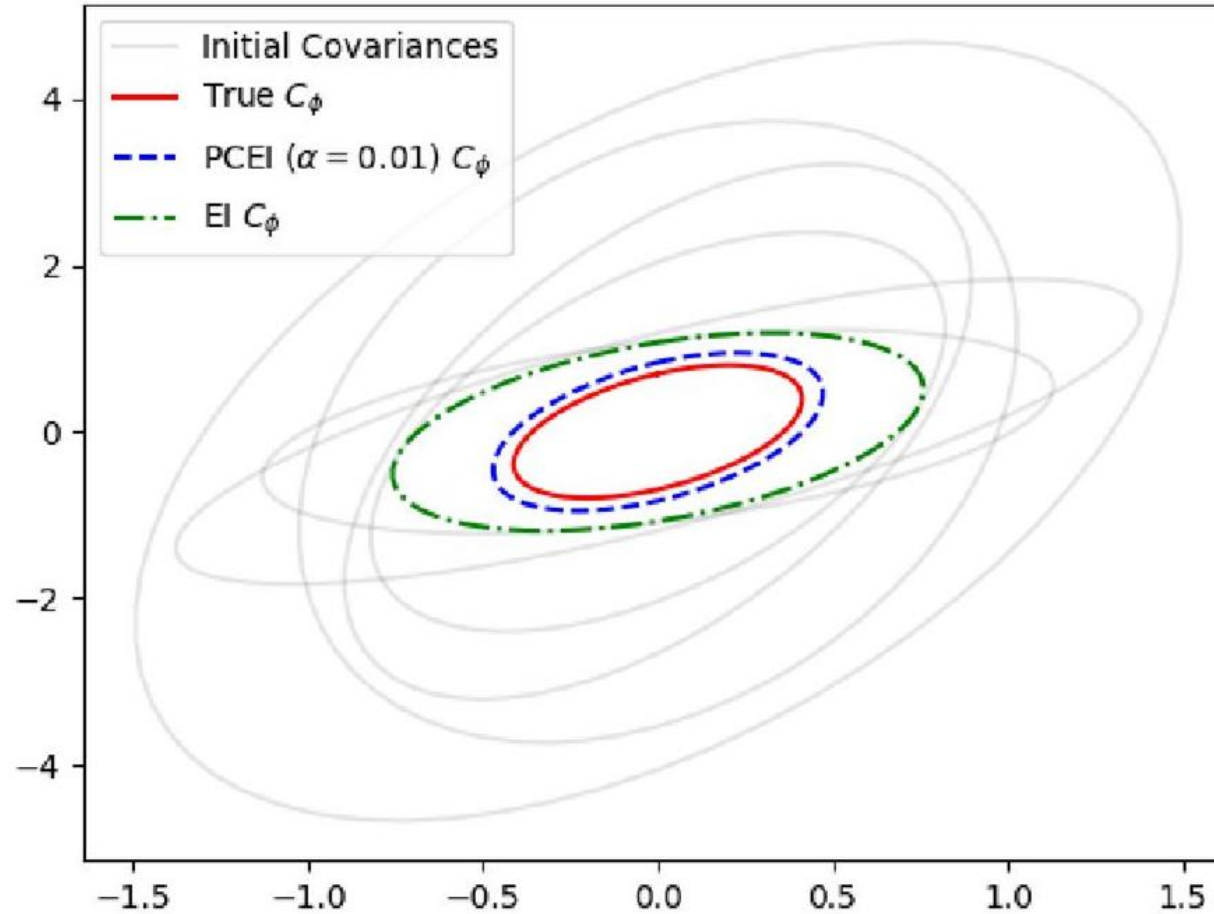
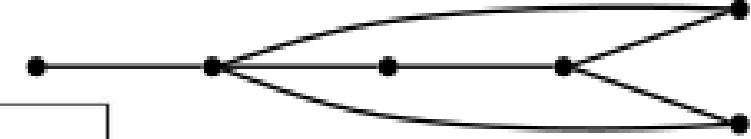
- Input 1 (1,1)
- Input 2 (1, $C_a$ )



Notice that as  $C_a$  approaches 1 EI's independent means asymptotically approach infinity.

# Results: Decentralized Network Convergence

- The randomly generated sensor topology is as follows...



- Probabilistic Constraint → less conservative fused covariances

# PC fusion applied to RF velocity estimation

- Test using multi-GNSS velocity estimation
- Differences in clocks between systems lead to unknown correlations
- Accuracy of covariance can be evaluated using ANEES (should be dimension of state)

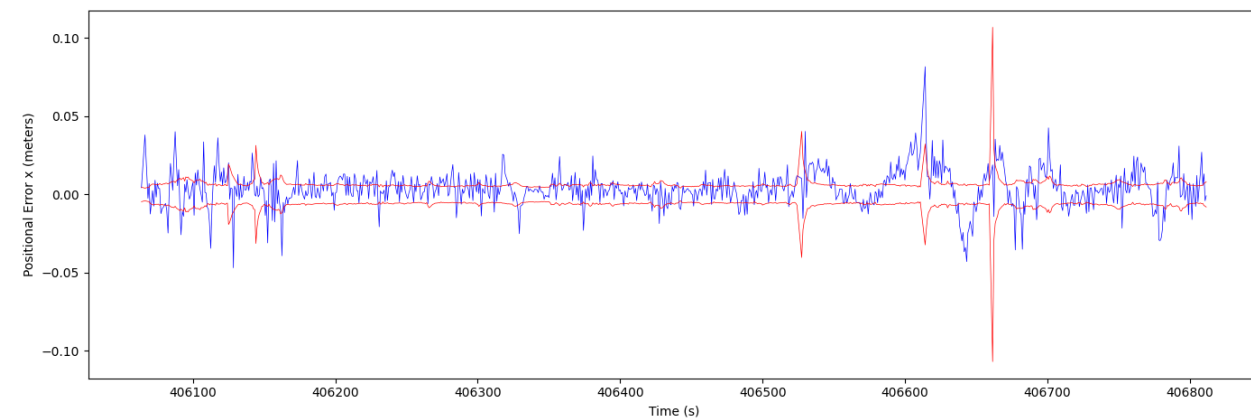
$$\lambda_i = e_i^\top C_i^{-1} e_i$$

$$ANEES = \frac{1}{N} \sum_{i=1}^N \lambda_i$$

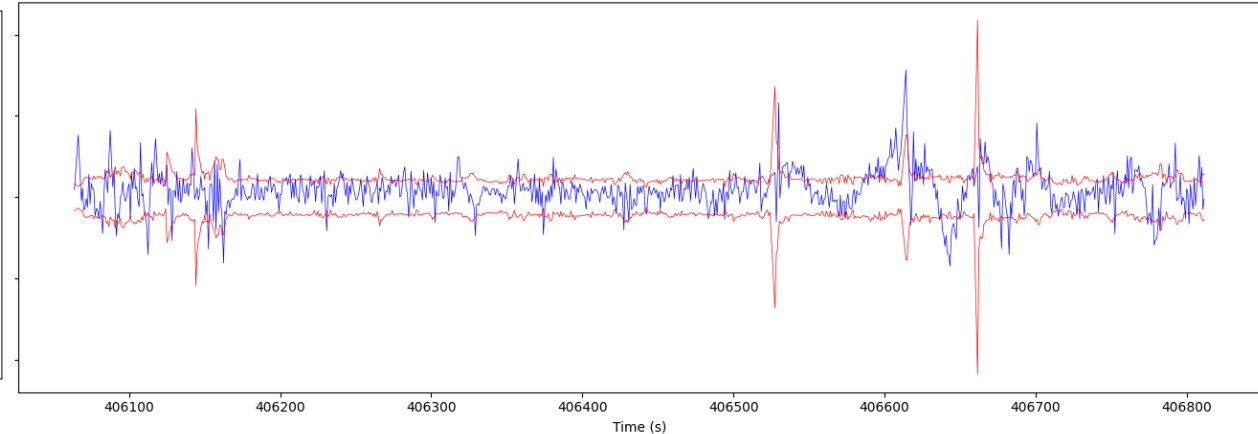


# Results with GNSS-based velocity estimation

|  | $MSE(*10^{-4})$ | $ANEES$ |
|--|-----------------|---------|
| Bayesian (naive)                               | 5.97            | 9.578   |
| Ellipsoidal Intersection                       | 149.13          | 2.888   |
| Covariance Intersection                        | 6.64            | 1.817   |
| Probabilistic Conservative ( $\alpha = 0.05$ ) | 6.45            | 3.088   |



Bayesian fusion errors (with 1-sigma lines), x velocity



PC fusion errors (with 1-sigma lines), x velocity

# Conclusions / Future Work

- Novel correlation-agnostic fusion technique was proposed (probabilistically constrained)
- Demonstrated improved performance in simulated network data and real GNSS velocity data
- Future Work:
  - Apply to real-world distributed network estimates
  - Apply to ground-based signals of opportunity